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(1, 2)*-πgη-CLOSED SETS IN BITOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce and investigate a new class of sets called (1, 2)*-πgη-closed and some new functions called (1, 2)*-πgη-continuous, almost (1, 2)*-πgη-continuous functions in bitopological spaces. Moreover we obtain the relationships among some existing closed sets like (1, 2)*-πgη-closed, (1, 2)*-semi-closed, (1, 2)*-α-closed and (1, 2)*-η-closed sets and their generalizations. Also we study some basic properties of (1, 2)*-πgη-closed sets. Further, we introduce (1, 2)*-πgη-neighbourhood and discuss some properties of (1, 2)*-πgη-neighbourhood.

KEYWORDS: *(1, 2)*-η-open, (1, 2)*-gη-closed, (1, 2)*-πgη-closed sets; (1, 2)*-πgη-continuous, almost (1, 2)*-πgη-continuous functions.*
2020 AMS Subject Classification: 54A05, 54A10, 54E55

1. INTRODUCTION

The study of bitopological space was first initiated by Kelly [**5**] in 1963. By using the topological notions, namely, semi-open, α-open and pre-open sets, many new bitopological sets are defined and studied by many topologists. In 2004, Ravi and Thivagar [**11**] studied the concept of stronger from of (1, 2)*-quotient mapping in bitopological spaces and also introduced the concepts of (1, 2)*-semi-open and (1, 2)*-α-open sets. In 2010, Arockiarani [**2**] introduced (1, 2)*-πgα-closed sets in bitopological spaces and studied some basic properties of (1, 2)*-πgα-closed sets. In 2010, K. Kayathri et al. [**4**] introduced and studied a new class of sets called regular (1, 2)*-g-closed sets and used it to obtain a new class of functions called (1, 2)*-rg-continuous, (1, 2)*-R-map, almost (1, 2)*-continuous and almost (1, 2)*-rg-closed functions in bitopological spaces and also obtained characterizations and preservation theorems for mildly (1, 2)*-normal spaces. In 2022, H. Kumar [**6**] introduced the concept of (1, 2)*-η-open sets and (1, 2)*-η-neighbourhood and; studied their properties. In 2022, H. Kumar [**7**] introduced the concept of (1, 2)*-generalized η-closed sets and studied some basic properties of (1, 2)*-gη-closed sets. In 2022, H. Kumar [**8**] introduced the concept of regular (1, 2)*-generalized η-closed sets and (1, 2)*-rgη-neighbourhood and; discussed their properties. Recently, H. Kumar [**9**] introduced and investigated some new functions called (1, 2)*-η-continuous, (1, 2)*-gη-continuous, (1, 2)*-rgη-continuous, almost (1, 2)*-η-continuous, almost (1, 2)*-gη-continuous, almost (1, 2)*-rgη-continuous, (1, 2)*-η-closed, (1, 2)*-gη-closed, (1, 2)*-rgη-closed, almost (1, 2)*-η-closed, almost (1, 2)*-gη-closed and almost (1, 2)*-rgη-closed functions in bitopological spaces and obtained characterizations and preservation theorems for mildly (1, 2)*-η-normal spaces.

2. PRELIMINARIES

Throughout the paper $(X, \Im_1, \Im_2)$, (Y, σ_1, σ_2) and $(Z, \wp_1, \wp_2)$ (or simply X, Y and Z) denote bitopological spaces.

Definition 2.1. Let S be a subset of X. Then S is said to be **$\Im_{1,2}$-open [11]** if $S = A \cup B$ where $A \in \Im_1$ and $B \in \Im_2$. The complement of a $\Im_{1,2}$-open set is **$\Im_{1,2}$-closed**.

Definition 2.2 [**11**]. Let S be a subset of X. Then
(i) the **$\Im_{1,2}$-closure** of S, denoted by $\Im_{1,2}$-cl(S), is defined as $\cap$ {F : S $\subset$ F and F is $\Im_{1,2}$-closed}; (ii) the **$\Im_{1,2}$-interior** of S, denoted by $\Im_{1,2}$-int(S), is defined as $\cup$ {F : F $\subset$ S and F is $\Im_{1,2}$-open}.

Note 2.3 [**11**]. Notice that $\Im_{1,2}$-open sets need not necessarily form a topology.

Definition 2.4. A subset A of a bitopological space $(X, \Im_1, \Im_2)$ is called
- **(i)** **regular (1, 2)*-open** [**11**] if A = $\Im_{1,2}$-int($\Im_{1,2}$-cl((A)).
- **(ii)** **(1, 2)*-π-open** [**2**] if A is the finite union of (1, 2)*-regular-open sets.

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(iii) **(1, 2)*-semi-open** [**11**] if $A = \Im_{1,2}\text{-cl}(\Im_{1,2}\text{-int}(A))$,
(iv) **(1 , 2)*-α-open** [**11**] if $A \subset \Im_{1,2}\text{-int}(\Im_{1,2}\text{-cl}(\Im_{1,2}\text{-int}(A)))$.
(v) **(1, 2)*-η-open** [**6**] if $A \subset \Im_{1,2}\text{-int}(\Im_{1,2}\text{-cl}(\Im_{1,2}\text{-int})(A)) \cup \Im_{1,2}\text{-cl}(\Im_{1,2}\text{–int}(A))$.

The complement of a regular (1, 2)*-open (resp. (1, 2)*-π-open, (1, 2)*-s-open, (1, 2)*-α-open, (1, 2)*-η-open) set is called **regular (1 , 2)*-closed** (resp. **(1, 2)*-π-closed**, **(1, 2)*-s-closed, (1, 2)*-α-closed, (1, 2)*-η-closed**).

The **(1, 2)*-s-closure (resp. (1, 2)*-α-closure, (1, 2)*-η-closure)** of a subset A of X is denoted by **(1, 2)*-s-cl(A) (resp. (1, 2)*-α-cl(A), (1, 2)*-η-cl(A))**, defined as the intersection of all (1, 2)*-s-closed (resp. (1, 2)*-α-closed, (1, 2)*-η-closed) sets containing A.

The family of all regular (1, 2)*-open (resp. regular (1, 2)*-closed, (1, 2)*-s-open, (1, 2)*-s-closed, (1, 2)*-α-open, (1, 2)*-α-closed, (1, 2)*-η-open, (1, 2)*-η-closed) sets in X is denoted by (1, 2)*-RO(X) (resp. (1, 2)*-RC(X), (1, 2)*-SO(X), (1, 2)*-SC(X), (1, 2)*-α-O(X), (1, 2)*-α-C(X), (1, 2)*-η-O(X), (1, 2)*-η-C(X)).

Remark 2.5. We have the following implications for the properties of subsets:

$$\begin{array}{ccccc}
\text{regular } (1,2)^*\text{-open} & \Rightarrow & (1,2)^*\text{-}\pi\text{-open} & \Rightarrow & \Im_{1,2}\text{-open} \\
\Downarrow & & \Downarrow & & \Downarrow \\
(1,2)^*\text{-}\eta\text{-open} & \Leftarrow & (1,2)^*\text{-s-open} & \Leftarrow & (1,2)^*\text{-}\alpha\text{-open}
\end{array}$$

Where none of the implications is reversible as can be seen from the following examples:

Example 2.6. Let X = {a, b, c, d}, $\Im_1 = \{\phi, \{a\}, X\}$ and $\Im_2 = \{\phi, \{b\}, \{a, b, c\}, X\}$. Then
(i) The $\Im_{1,2}$-open sets are : ϕ, {a}, {b}, {a, b}, {a, b, c}, X.
(ii) The regular (1, 2)*-open sets are : ϕ, {a}, {b}, X.
(iii) The (1, 2)*-π-open sets are : ϕ, {a}, {b}, {a, b}, X.
(iv) The (1, 2)*-semi open sets are : ϕ, {a}, {b}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X.
(v) The (1, 2)*-α-open sets are : ϕ, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}, X
(vi) The (1, 2)*-η-open sets are : ϕ, {a}, {b}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X.

Example 2.7. Let X = {a, b, c}, $\Im_1 = \{\phi, \{b\}, X\}$ and $\Im_2 = \{\phi, \{c\}, X\}$. Then
(i) The $\Im_{1,2}$-open sets are : ϕ, {b}, {c}, {b, c}, X}.
(ii) The regular (1, 2)*-open sets are : ϕ, {b}, {c}, X.
(iii) The (1, 2)*-π-open sets are : ϕ, {b}, {c}, {b, c}, X.
(iv) The (1, 2)*-semi-open sets are : ϕ, X, {b}, {c}, {a, b}, {a, c}, {b, c}.
(v) The (1, 2)*-α-open sets are : ϕ, X, {b}, {c}, {b, c}.
(vi) The (1, 2)*-η-open sets are : ϕ, {b}, {c}, {a, b}, {a, c}, {b. c}, X.

Example 2.8. Let X = {a, b, c, d} with $\Im_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$ and $\Im_2 = \{\phi, X, \{c\}, \{a, c, d\}\}$. Then
(i) The $\Im_1\Im_2$-open sets are : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}, {a, c, d}, {b, c, d}.
(ii) The regular (1, 2)*-open sets are : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c, d}, {b, c, d}.
(iii) The (1, 2)*-π-open sets are : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}, {a, c, d}, {b, c, d}.
(iv)The (1, 2)*-semi open sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(v) The (1, 2)*-α-open sets are : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}, {a, c, d}, {b, c, d}.
(vi) The (1, 2)*-η-open sets are : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.

3. (1, 2)*-RGη-CLOSED SETS IN BITOPOLOGICAL SPACES

Definition 3.1. A subset A of a bitopological space $(X, \Im_1, \Im_2)$ is called
(i) **(1, 2)*-generalized closed** (briefly **(1, 2)*-g-closed**) [**12**] if $\Im_{1,2}\text{-cl}(A) \subset U$ whenever $A \subset U$ and U is $\Im_{1,2}$-open in X.
(ii) **(1, 2)*-π-generalized closed** (briefly **(1, 2)*-πg-closed**) [13] if $\Im_{1,2}\text{-cl}(A) \subset U$ whenever $A \subset U$ and U is (1, 2)*-π-open in X.
(iii) **regular (1, 2)*-generalized closed** (briefly **(1, 2)*-rg-closed**) [**4**] if $\Im_{1,2}\text{-cl}(A) \subset U$ whenever $A \subset U$ and $U \in (1, 2)^*\text{-RO}(X)$.
(iv) **(1, 2)*-α-generalized closed** (briefly **(1, 2)*-αg-closed**) [**11**] if $(1, 2)^*\text{-}\alpha\text{-cl}(A) \subset U$ whenever $A \subset U$ and U is $\Im_{1,2}$-open in X.
(v) **(1, 2)*-π-generalized α-closed** (briefly **(1, 2)*-πgα-closed**) [**2**] if $(1, 2)^*\text{-}\alpha\text{-cl}(A) \subset U$ whenever $A \subset U$ and U is (1, 2)*-π-open in X.

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(vi) **regular (1, 2)*-generalized α-closed** (briefly **(1, 2)*-rgα-closed**) [**14**] if (1, 2)*- α-cl(A) $\subset$ U whenever A $\subset$ U and U $\in$ (1, 2)*-RO(X).
(vii) **(1, 2)*-generalized semi-closed** (briefly **(1, 2)*-gs-closed**) [**13**] if (1, 2)*-α-cl(A) $\subset$ U whenever A $\subset$ U and U is $\Im_{1,2}$-open in X.
(viii) **(1, 2)*-π-generalized semi-closed** (briefly **(1, 2)*-πgs-closed**) [**14**] if (1, 2)*-α-cl(A) $\subset$ U whenever A $\subset$ U and U is (1, 2)*-π-open in X.
(ix) **regular (1, 2)*-generalized semi-closed** (briefly **(1, 2)*-rgs-closed**) [**12**] if (1, 2)*- α-cl(A) $\subset$ U whenever A $\subset$ U and U $\in$ (1, 2)*-RO(X).
(x) **(1, 2)*-generalized η-closed** (briefly **(1, 2)*-gη-closed**) [7] if (1, 2)*-η-cl(A) $\subset$ U whenever A $\subset$ U and U is $\Im_{1,2}$-open in X.
(xi) **(1, 2)*-generalized η-closed** (briefly **(1, 2)*-πgη-closed**) if (1, 2)*-η-cl(A) $\subset$ U whenever A $\subset$ U and U is (1, 2)*-π-open in X.
(xii) **regular (1, 2)*-generalized η-closed** (briefly **(1, 2)*-rgη-closed**) [**8**] if (1, 2)*-η-cl(A) $\subset$ U whenever A $\subset$ U and U $\in$ (1, 2)*-RO(X).

The complement of a (1, 2)*-g-closed (resp. (1, 2)*-πg-closed, (1, 2)*-rg-closed, (1, 2)*-αg-closed, (1, 2)*-πgα-closed, (1, 2)*-rgα-closed, (1, 2)*-gs-closed, (1, 2)*-πgs-closed, (1, 2)*-rgs-closed, (1, 2)*-gη-closed, (1, 2)*-πgη-closed, (1, 2)*-rgη-closed) set is called **(1, 2)*-g-open** (resp. **(1, 2)*-πg-open, (1, 2)*-rg-open**, **(1, 2)*-αg-open**, **(1, 2)*-πgα-open, (1, 2)*-rgα-open, (1, 2)*-gs-open**, **(1, 2)*-πgs-open, (1, 2)*-rgs-open (1, 2)*-gη-open**, **(1, 2)*-πgη-open, (1, 2)*-rgη-open**).

We denote the set of all (1, 2)*-rgη-closed (resp. (1, 2)*-rgη-open) sets in (X, $\Im_1$, $\Im_2$) by (1, 2)*-**rgη-C(X)** (resp. **rgη-O(X)**).

Theorem 3.2. Every $\Im_{1,2}$-closed set is πgη-closed.
Proof. Let A be any $\Im_{1,2}$-closed set in (X, $\Im_1$, $\Im_2$) and A $\subset$ U, where U is (1, 2)*-π-open set. So (1, 2)*-cl(A) = A . Since every $\Im_{1,2}$-closed set is (1, 2)*-η-closed, so (1, 2)*-η-cl(A) $\subset$ (1, 2)*-cl(A) = A. Therefore, (1, 2)*-η-cl(A) $\subset$ A $\subset$ U. Hence A is (1, 2)*-πgη-closed set.

Theorem 3.3. Every (1, 2)*-g-closed set is (1, 2)*-rgη-closed.
Proof. Let A be any (1, 2)*-g-closed set in (X, $\Im_1$, $\Im_2$) such that (1, 2)*-cl(A) $\subset$ U whenever A $\subset$ U, where U is (1, 2)*-π-open set, since every (1, 2)*-π-open set is $\Im_{1,2}$-open. So (1, 2)*-η-cl(A) $\subset$ (1, 2)*-cl(A) $\subset$ U. Therefore (1, 2)*-η-cl(A) $\subset$ U. Hence A is (1, 2)*-πgη-closed set.

Theorem 3.4. Every (1, 2)*-πg-closed set is (1, 2)*-πgη-closed.
Proof. Let A be any (1, 2)*-πg-closed set in (X, $\Im_1$, $\Im_2$) such that (1, 2)*-cl(A) $\subset$ U whenever A $\subset$ U, where U is (1, 2)*-π-open set. So (1, 2)*-η-cl(A) $\subset$ (1, 2)*-cl(A) $\subset$ U. Therefore (1, 2)*-η-cl(A) $\subset$ U. Hence A is (1, 2)*-πgη-closed set.

Theorem 3.5. Every (1, 2)*-α-closed set is (1, 2)*-πgη-closed.
Proof. Let A be any (1, 2)*-α-closed set in (X, $\Im_1$, $\Im_2$) and A $\subset$ U, where U is (1, 2)*-π-open set. Since every (1, 2)*-α-closed set is (1, 2)*-η-closed, so (1, 2)*-η-cl(A) $\subset$ (1, 2)*-α-cl(A) = A. Therefore (1, 2)*-η-cl(A) $\subset$ A $\subset$ U. Hence A is (1, 2)*-πgη-closed set.

Theorem 3.6. Every (1, 2)*-αg-closed set is (1, 2)*-πgη-closed.
Proof. Let A be any (1, 2)*-αg-closed set in (X, $\Im_1$, $\Im_2$) such that (1, 2)*-α-cl(A) $\subset$ U whenever A $\subset$ U, where U is (1, 2)*-π-open set, since every (1, 2)*-π-open set is $\Im_{1,2}$-open. Given that A is (1, 2)*-αg-closed set such that (1, 2)*-α-cl(A) $\subset$ U. But we have (1, 2)*-η-cl(A) $\subset$ (1, 2)*-α-cl(A) $\subset$ U. Therefore (1, 2)*-η-cl(A) $\subset$ U. Hence A is (1, 2)*-πgη-closed set.
Theorem 3.7. Every (1, 2)*-πgα-closed set is (1, 2)*-πgη-closed.
Proof. Let A be any (1, 2)*-πgα-closed set in (X, $\Im_1$, $\Im_2$) such that (1, 2)*-α-cl(A) $\subset$ U whenever A $\subset$ U, where U is (1, 2)*-π-open set. Given that A is (1, 2)*-πgα-closed set such that (1, 2)*-α-cl(A) $\subset$ U. But we have (1, 2)*-η-cl(A) $\subset$ (1, 2)*-α-cl(A) $\subset$ U. Therefore (1, 2)*-η-cl(A) $\subset$ U. Hence A is (1, 2)*-πgη-closed set.

Theorem 3.8. Every (1, 2)*-semi-closed set is (1, 2)*-πgη-closed.
Proof. Let A be any (1, 2)*-semi-closed set in (X, $\Im_1$, $\Im_2$) and A $\subset$ U, where U is (1, 2)*-π-open set. Since every (1, 2)*-semi-closed set is (1, 2)*-η-closed, so (1, 2)*-η-cl(A) $\subset$ (1, 2)*-s-cl(A) = A. Therefore (1, 2)*-η-cl(A) $\subset$ A $\subset$ U. Hence A is (1, 2)*-πgη-closed set.

Theorem 3.9. Every (1, 2)*-gs-closed set is (1, 2)*-πgη-closed.
Proof. Let A be any (1, 2)*-gs-closed set in (X, $\Im_1$, $\Im_2$) such that (1, 2)*-s-cl(A) $\subset$ U whenever A $\subset$ U, where U is (1, 2)*-π-open set, since every (1 , 2)*-π-open set is $\Im_{1,2}$-open. Given that A is (1, 2)*-gs-closed set such that (1, 2)*-s-cl(A) $\subset$ U. But we have (1, 2)*-η-cl(A) $\subset$ (1, 2)*-s-cl(A) $\subset$ U. Therefore (1, 2)*-η-cl(A) $\subset$ U. Hence A is (1, 2)*-πgη-closed set.

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Theorem 3.10. Every $(1, 2)^*$-πgs-closed set is $(1, 2)^*$-πgη-closed.
Proof. Let A be any $(1, 2)^*$-πgs-closed set in $(X, \Im_1, \Im_2)$ such that $(1, 2)^*$-s-cl(A) $\subset$ U whenever A $\subset$ U, where U is $(1, 2)^*$-π-open set. Given that A is $(1, 2)^*$-gs-closed set such that $(1, 2)^*$-s-cl(A) $\subset$ U. But we have $(1, 2)^*$-η-cl(A) $\subset$ $(1, 2)^*$-s-cl(A) $\subset$ U. Therefore $(1, 2)^*$-η-cl(A) $\subset$ U. Hence A is $(1, 2)^*$-πgη-closed set.

Theorem 3.11. Every $(1, 2)^*$-η-closed set is $(1, 2)^*$-πgη-closed.
Proof. Let A be any $(1, 2)^*$-η-closed set in $(X, \Im_1, \Im_2)$ and A $\subset$ U, where U is $(1, 2)^*$-π-open set. Since A is $(1, 2)^*$-η-closed. Therefore $(1, 2)^*$-η-cl(A) = A $\subset$ U. Hence A is $(1, 2)^*$-πgη-closed set.

Theorem 3.12. Every $(1, 2)^*$-gη-closed set is $(1, 2)^*$-rgη-closed.
Proof. Let A be any $(1, 2)^*$-gη-closed set in $(X, \Im_1, \Im_2)$ such that $(1, 2)^*$-η-cl(A) $\subset$ U whenever A $\subset$ U, where U is $(1, 2)^*$-π-open set, since every $(1, 2)^*$-π-open set is $\Im_{1,2}$-open. Given that A is $(1, 2)^*$-gη-closed set such that $(1, 2)^*$-η-cl(A) $\subset$ U. Therefore $(1, 2)^*$-η-cl(A) $\subset$ U. Hence A is $(1, 2)^*$-πgη-closed set.

Corollary 3.13. Every regular $(1, 2)^*$-closed set is $(1, 2)^*$-πgη-closed.
Proof. Since every regular $(1, 2)^*$-closed set is $\Im_{1,2}$-closed. So by Theorem 3.2, every $\Im_{1,2}$-closed set is $(1, 2)^*$-πgη-closed.

Corollary 3.14. Every $(1, 2)^*$-π-closed set is $(1, 2)^*$-πgη-closed.
Proof. Since every $(1, 2)^*$-π-closed set is $\Im_{1,2}$-closed. So by Theorem 3.2, every $\Im_{1,2}$-closed set is $(1, 2)^*$-πgη-closed.

Remark 3.15. We have the following implications for the properties of subsets:

$$\begin{array}{ccccccc}
(1,2)^*\text{-}\pi\text{-closed} & \Leftarrow & \text{regular } (1,2)^*\text{-closed} & & & & \\
\Downarrow & & \Downarrow & & & & \\
\Im_{1,2}\text{-closed} & \Rightarrow & (1,2)^*\text{-g-closed} & \Rightarrow & (1,2)^*\text{-}\pi\text{g-closed} & \Rightarrow & (1,2)^*\text{-rg-closed} \\
\Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\
(1,2)^*\text{-}\alpha\text{-closed} & \Rightarrow & (1,2)^*\text{-g}\alpha\text{-closed} & \Rightarrow & (1,2)^*\text{-}\pi\text{g}\alpha\text{-closed} & \Rightarrow & (1,2)^*\text{-rg}\alpha\text{-closed} \\
\Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\
(1,2)^*\text{-s-closed} & \Rightarrow & (1,2)^*\text{-gs-closed} & \Rightarrow & (1,2)^*\text{-}\pi\text{gs-closed} & \Rightarrow & (1,2)^*\text{-rg}\alpha\text{-closed} \\
\Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\
(1,2)^*\text{-}\eta\text{-closed} & \Rightarrow & (1,2)^*\text{-g}\eta\text{-closed} & \Rightarrow & (1,2)^*\text{-}\pi\text{g}\eta\text{-closed} & \Rightarrow & (1,2)^*\text{-rg}\eta\text{-closed}
\end{array}$$

Where none of the implications is reversible as can be seen from the following examples:

Example 3.16. Let X = {a, b, c, d} with $\Im_1$ = {ϕ, X, {a}, {b}, {a, b}, {b, c, d}} and $\Im_2$ = {ϕ, X, {c}, {a, c, d}}. Then
(i) regular $(1, 2)^*$-closed : ϕ, X, {a}, {b}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(ii) $(1, 2)^*$-π-closed : ϕ, X, {a}, {b}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(iii) $\Im_{1,2}$-closed sets : ϕ, X, {a}, {b}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(iv) $(1, 2)^*$-g-closed sets : ϕ, X, {a}, {b}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(v) $(1, 2)^*$-πg-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(vi) $(1, 2)^*$-rg-closed sets : ϕ, X, {a}, {b}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(vii) $(1, 2)^*$-α-closed sets : ϕ, X, {a}, {b}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(viii) $(1, 2)^*$-αg-closed sets : ϕ, X, {a}, {b}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(ix) $(1, 2)^*$-πgα-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(x) $(1, 2)^*$-rgα-closed sets : ϕ, X, {a}, {b}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(xi) $(1, 2)^*$-s-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(xii) $(1, 2)^*$-gs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(xiii) $(1, 2)^*$-πgs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(xiv) $(1, 2)^*$-rgs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(xv) $(1, 2)^*$-η-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(xvi) $(1, 2)^*$-gη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(xvii) $(1, 2)^*$-πgη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.

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(xviii) $(1, 2)^*$-rgη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.

Example 3.17. Let X = {a, b, c} with $\Im_1$ = {ϕ, X, {b}} and $\Im_2$ = {ϕ, X, {c}}. Then
(i) regular $(1, 2)^*$-closed : ϕ, X, {a, b}, {a, c}.
(ii) $(1, 2)^*$-π-closed : ϕ, X, {a}, {a, b}, {a, c}.
(iii) $\Im_{1,2}$-closed sets : ϕ, X, {a}, {a, b}, {a, c}.
(iv) $(1, 2)^*$-g-closed sets : ϕ, X, {a}, {a, b}, {a, c}.
(v) $(1, 2)^*$-πg-closed sets : ϕ, X, {a}, {a, b}, {a, c}.
(vi) $(1, 2)^*$-rg-closed sets : ϕ, X, {a}, {a, b}, {a, c}, {b, c}.
(vii) $(1, 2)^*$-α-closed sets : ϕ, X, {a}, {a, b}, {a, c}.
(viii) $(1, 2)^*$-αg-closed sets : ϕ, X, {a}, {a, b}, {a, c}.
(ix) $(1, 2)^*$-πgα-closed sets : ϕ, X, {a}, {a, b}, {a, c}.
(x) $(1, 2)^*$-rgα-closed sets : ϕ, X, {a}, {a, b}, {a, c}, {b, c}.
(xi) $(1, 2)^*$-s-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}.
(xii) $(1, 2)^*$-gs-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}.
(xiii) $(1, 2)^*$-πgs-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}.
(xiv) $(1, 2)^*$-rgs-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}.
(xv) $(1, 2)^*$-η-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}.
(xvi) $(1, 2)^*$-gη-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}.
(xvii) $(1, 2)^*$-πgη-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}.
(xviii) $(1, 2)^*$-rgη-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}.

Example 3.18. Let X = {a, b, c, d} with $\Im_1$ = {ϕ, X, {a}} and $\Im_2$ = {ϕ, X, {b}, {a, b, c}}. Then
(i) regular $(1, 2)^*$-closed : ϕ, X, {a, c, d}, {b, c, d}.
(ii) $(1, 2)^*$-π-closed : ϕ, X, {c, d}, {a, c, d}, {b, c, d}.
(iii) $\Im_{1,2}$-closed sets : ϕ, X, {d}, {c, d}, {a, c, d}, {b, c, d}.
(iv) $(1, 2)^*$-g-closed sets : ϕ, X, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(v) $(1, 2)^*$-πg-closed sets : ϕ, X, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(vi) $(1, 2)^*$-rg-closed sets : ϕ, X, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(vii) $(1, 2)^*$-α-closed sets : ϕ, X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}.
(viii) $(1, 2)^*$-αg-closed sets : ϕ, X, {c}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(ix) $(1, 2)^*$-πgα-closed sets : ϕ, X, {a}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(x) $(1, 2)^*$-rgα-closed sets : ϕ, X, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(xi) $(1, 2)^*$-s-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}.
(xii) $(1, 2)^*$-gs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(xiii) $(1, 2)^*$-πgs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(xiv) $(1, 2)^*$-rgs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(xv) $(1, 2)^*$-η-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}.
(xvi) $(1, 2)^*$-gη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(xvii) $(1, 2)^*$-πgη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(xviii) $(1, 2)^*$-rgη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.

Example 3.19. Let X = {a, b, c, d} with $\Im_1$ = {ϕ, X, {a}, {b}, {a, b}, {a, b, c}} and $\Im_2$ = {ϕ, X, {a, b, d}}. Then
(i) regular $(1, 2)^*$-closed : ϕ, X, {a, c, d}, {b, c, d}.
(ii) $(1, 2)^*$-π-closed : ϕ, X, {c, d}, {a, c, d}, {b, c, d}.
(iii) $\Im_{1,2}$-closed sets : ϕ, X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}.
(iv) $(1, 2)^*$-g-closed sets : ϕ, X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}.
(v) $(1, 2)^*$-πg-closed sets : ϕ, X, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(vi) $(1, 2)^*$-rg-closed sets : ϕ, X, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(vii) $(1, 2)^*$-α-closed sets : ϕ, X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}.
(viii) $(1, 2)^*$-αg-closed sets : ϕ, X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}.

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(ix) $(1, 2)^*$-πgα-closed sets : ϕ, X, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(x) $(1, 2)^*$-rgα-closed sets : ϕ, X, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(xi) $(1, 2)^*$-s-closed sets : ϕ, X, {a}, {b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}.
(xii) $(1, 2)^*$-gs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}.
(xiii) $(1, 2)^*$-πgs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(xiv) $(1, 2)^*$-rgs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(xv) $(1, 2)^*$-η-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}.
(xvi) $(1, 2)^*$-gη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}.
(xvii) $(1, 2)^*$-πgη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(xviii) $(1, 2)^*$-rgη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.

4. PROPERTIES OF $(1, 2)^*$-π-GENERALIZED η-CLOSED SETS

In this section, we study some basic properties of $(1, 2)^*$-πgη-closed sets. Also, we introduce $(1, 2)^*$-πgη-neighborhood (shortly $(1, 2)^*$-πgη-nbd in bitopological spaces by using the notion of $(1, 2)^*$-πgη-open sets. We prove that every nbd of x in $(X, \Im_1, \Im_2)$ is $(1, 2)^*$-πgη-nbhd of x but not conversely.

Theorem 4.1. The union of two $(1, 2)^*$-πgη-closed sets of $(X, \Im_1, \Im_2)$ need not be an $(1, 2)^*$-rgη-closed set of $(X, \Im_1, \Im_2)$.
Proof. This can be seen from the following example.

Example 4.2. Let X = {a, b, c, d} with $\Im_1$ = {ϕ, X, {a}, {b}, {a, b}, {b, c, d}} and $\Im_2$ = {ϕ, X, {c}, {a, c, d}}. Let A = {a} and B = {c} be $(1, 2)^*$-πgη-closed sets but A ∪ B = {a, c} is not an $(1, 2)^*$-πgη-closed set.

Theorem 4.3. The intersection of two $(1, 2)^*$-πgη-closed-sets in $(X, \Im_{1,} \Im_2)$ is also a $(1, 2)^*$-rgη-closed set in $(X, \Im_{1,} \Im_2)$.
Proof. Easy to proof.

Theorem 4.4. If a subset A is $(1, 2)^*$-πgη-closed, then $(1, 2)^*$-η-cl(A) – A does not contain any non-empty $(1, 2)^*$-π-closed set.
Proof. Suppose that A is $(1, 2)^*$-πgη-closed. Let F be an $(1, 2)^*$-π-closed subset of $(1, 2)^*$-η-cl(A) – A. Then F ⊂ [$(1, 2)^*$-η-cl(A) ∩ (X – A)] and so A ⊂ [X – F]. But A is $(1, 2)^*$-πgη-closed. Therefore $(1, 2)^*$-η-cl(A) ⊂ [X – F]. Consequently, F ⊂ [X – $(1, 2)^*$-η-cl(A)]. We already have F ⊂ $(1, 2)^*$-η-cl(A). Hence F ⊂ [$(1, 2)^*$-η-cl(A) ∩ X – $(1, 2)^*$-η-cl(A)] = ϕ. Thus F = ϕ. Therefore $(1, 2)^*$-η-cl(A) – A contains no non-empty $(1, 2)^*$-π-closed set.

Example 4.5. The converse of Theorem 4.4 is not true. This can be seen from the following example.

Example 4.6. Let X = {a, b, c, d} with $\Im_1$ = {ϕ, X, {a}, {b}, {a, b}, {a, b, c}} and $\Im_2$ = {ϕ, X, {a, b, d}}. Let A = {a, b, c}. We have that $(1, 2)^*$-η-cl(A) – A = X – {a, b, c} = {d} does not contain any non-empty $(1, 2)^*$-π-closed set. However, A is $(1, 2)^*$-πgη-closed in X.

Theorem 4.7. For an element x ∈ $(X, \Im_1, \Im_2)$, the set $(X, \Im_1, \Im_2)$ – {x} is $(1, 2)^*$- πgη-closed or $(1, 2)^*$-π-open.
Proof. Suppose $(X, \Im_1, \Im_2)$ – {x} is not $(1, 2)^*$-π-open set. Then $(X, \Im_1, \Im_2)$ is the only $(1, 2)^*$- π-open set containing $(X, \Im_1, \Im_2)$ – {x}. This implies $(1, 2)^*$-η-cl($(X, \Im_1, \Im_2)$ – {x}) ⊂ $(X, \Im_1, \Im_2)$. Hence $(X, \Im_1, \Im_2)$ – {x} is $(1, 2)^*$-πgη-closed set in $(X, \Im_1, \Im_2)$.

Theorem 4.8. Let A be a $(1, 2)^*$-πgη-closed subset of X. If A ⊂ B ⊂ $(1, 2)^*$-η-cl(A), then B is also $(1, 2)^*$-πgη-closed in X.
Proof. Let U ∈ $(1, 2)^*$-πgη-O(X) with B ⊂ U. Then A ⊂ U. Since A is $(1, 2)^*$-πgη-closed, $(1, 2)^*$-η-cl(A) ⊂ U. Also, since B ⊂ $(1, 2)^*$-η-cl(A), $(1, 2)^*$-η-cl(B) ⊂ $(1, 2)^*$-η-cl(A) ⊂ U. Hence B is also $(1, 2)^*$-πgη-closed subset of X.

Remark 4.9. The converse of the **Theorem 4.8** need not be true in general. Consider the bitopological space $(X, \Im_1, \Im_2)$ where X = {a, b, c, d, e} with topology $\Im_1$ = {ϕ, {a, b}, {a, b, c, d}, X}, $\Im_2$ = {ϕ, {c, d}, {a, b, c, d}, X}, Let A = {b} and B = {b, c}. Then A and B are $(1, 2)^*$- πgη-closed sets in $(X, \Im_1, \Im_2)$ such that A ⊂ B but B ⊄ $(1, 2)^*$-η-cl(A) = {a, b}.

Theorem 4.10. Let A be a $(1, 2)^*$-πgη-closed in $(X, \Im_1, \Im_2)$. Then A is $(1, 2)^*$-η-closed if and only if $(1, 2)^*$-η-cl(A) – A is an $(1, 2)^*$-π-open.

Proof. Suppose A is a $(1, 2)^*$-η-closed in $(X, \Im_1, \Im_2)$. Then $(1, 2)^*$-η-cl(A) = A and so $(1, 2)^*$-η cl(A) – A = ϕ, which is $(1, 2)^*$-π-open in $(X, \Im_1, \Im_2)$.

Conversely, suppose $(1, 2)^*$-η-cl(A) – A is an $(1, 2)^*$-π-open set in $(X, \Im_1, \Im_2)$. Since A is $(1, 2)^*$- πgη-closed, by **Theorem 4.4** $(1, 2)^*$-η-cl(A) – A does not contain any nonempty $(1, 2)^*$-π-open in $(X, \Im_1, \Im_2)$. Then $(1, 2)^*$-η-cl(A) – A = ϕ. Hence A is (1, 2)*-η-closed set in $(X, \Im_1, \Im_2)$.

Theorem 4.11. If A is $(1, 2)^*$-π-open and $(1, 2)^*$-πgη-closed, then A is $(1, 2)^*$-πgη-closed set in $(X, \Im_1, \Im_2)$.
Proof. Let U be any $(1, 2)^*$-π-open set in $(X, \Im_1, \Im_2)$ such that $A \subset U$. Since A is $(1, 2)^*$-open and $(1, 2)^*$-πgη-closed, we have $(1, 2)^*$-η-cl(A) $\subset$ A. Then $(1, 2)^*$-η-cl(A) $\subset A \subset U$. Hence A is $(1, 2)^*$-πgη-closed set in $(X, \Im_1, \Im_2)$.

Theorem 4.12. If a subset A of bitopological space $(X, \Im_1, \Im_2)$ is both $(1, 2)^*$-π-open and $(1, 2)^*$- πgη-closed, then it is $(1, 2)^*$-η-closed.
Proof. Suppose a subset A of bitopological space $(X, \Im_1, \Im_2)$ is both $(1, 2)^*$-π-open and $(1, 2)^*$- πgη-closed. Now $A \subset A$. Then $(1, 2)^*$-η-cl(A) $\subset$ A. Hence A is $(1, 2)^*$-η-closed.

Corollary 4.13. Let A be $(1, 2)^*$-π-open and $(1, 2)^*$-πgη-closed subset in $(X, \Im_1, \Im_2)$. Suppose that F is $(1, 2)^*$-η-closed set in $(X, \Im_1, \Im_2)$. Then $A \cap F$ is an $(1, 2)^*$-πgη-closed set in $(X, \Im_1, \Im_2)$.
Proof. Let A be a $(1, 2)^*$-π-open and $(1, 2)^*$-πgη-closed subset in $(X, \Im_1, \Im_2)$ and F be closed. By **Theorem 4.12**, A is $(1, 2)^*$-η-closed. So $A \cap F$ is a $(1, 2)^*$-η-closed and hence $A \cap F$ is $(1, 2)^*$- πgη-closed set in $(X, \Im_1, \Im_2)$.

Theorem 4.14. If A is both open and $(1, 2)^*$-g-closed set in $(X, \Im_1, \Im_2)$, then it is $(1, 2)^*$-πgη-closed set in $(X, \Im_1, \Im_2)$.
Proof. Let A be an open and $(1, 2)^*$-g-closed set in $(X, \Im_1, \Im_2)$. Let $A \subset U$ and let U be a $(1, 2)^*$-π-open set in $(X, \Im_1, \Im_2)$. Now $A \subset A$. By hypothesis $(1, 2)^*$-η-cl(A) $\subset$ A. That is $(1, 2)^*$-η-cl(A) $\subset$ U. Thus A is $(1, 2)^*$-πgη-closed in $(X, \Im_1, \Im_2)$.

Theorem 4.15. A set A is $(1, 2)^*$-πgη-open if and only if the following condition holds:
F $\subset$ $(1, 2)^*$-η-int(A) whenever F is $(1, 2)^*$-π-closed and $F \subset A$.
Proof. Suppose the condition holds. Put [X – A] = B. Suppose that $B \subset U$ where U is $(1, 2)^*$-π-open. Now $X - A \subset U$ implies F = [X – U] $\subset$ A and F is $(1, 2)^*$-π-closed, which implies F $\subset$ $(1, 2)^*$-η-int(A). Also F $\subset$ $(1, 2)^*$-η-int(A) implies [X – $(1, 2)^*$-η-int(A)] $\subset$ [X – F] = U. This implies [X – ($(1, 2)^*$-η-int(X – B))] $\subset$ U. Therefore [X – ($(1, 2)^*$-η-int(X – B))] $\subset$ U or equivalently $(1, 2)^*$-η-cl(B) $\subset$ U. Thus B is $(1, 2)^*$-πgη-closed. Hence A is $(1, 2)^*$-πgη-open.

Conversely, suppose that A is $(1, 2)^*$-πgη-open, $F \subset A$ and F is $(1, 2)^*$-π-closed. Then [X – F] is $(1, 2)^*$-π-open. Then (X – A) $\subset$ (X – F). Hence $(1, 2)^*$-η-cl(X – A) $\subset$ (X – F) because (X – A) is $(1, 2)^*$-πgη-closed. Therefore F $\subset$ (X – $(1, 2)^*$-η-cl(X – A)) = $(1, 2)^*$-η-int(A).

Definition 4.16. Let $(X, \Im_1, \Im_2)$ be a bitopological space and let $x \in (X, \Im_1, \Im_2)$. A subset N of $(X, \Im_1, \Im_2)$ is said to be a **$(1, 2)^*$-πgη-neighbourhood** (briefly **$(1, 2)^*$-πgη-nbd**) of x iff there exists an $(1, 2)^*$-πgη-open set G such that $x \in G \subset N$.

Definition 4.17. A subset N of a bitopological space $(X, \Im_1, \Im_2)$, is called a **$(1, 2)^*$-πgη-nbd** of $A \subset (X, \Im_1, \Im_2)$ iff there exists a $(1, 2)^*$-πgη-open set G such that $A \subset G \subset N$.

Theorem 4.18. Every nbd N of $x \in (X, \Im_1, \Im_2)$ is a $(1, 2)^*$-πgη-nbd of $(X, \Im_1, \Im_2)$.
Proof. Let N be a nbd of point $x \in (X, \Im_1, \Im_2)$. To prove that N is a $(1, 2)^*$-rgη-nbd of x. By definition of nbd, there exists an open set G such that $x \in G \subset N$. As every open set is $(1, 2)^*$- πgη-open such that $x \in G \subset N$. Hence N is $(1, 2)^*$-πgη-nbd of x.

Remark 4.19. In general, a $(1, 2)^*$-πgη-nbd N of $x \in (X, \Im_1, \Im_2)$ need not be a nbd of x in $(X, \Im_1, \Im_2)$, as seen from the following example.

Example 4.20. Let X = {a, b, c, d, e} with topology $\Im_1$ = {ϕ, {a, b}, {a, b, c, d}, X} and $\Im_2$ = {ϕ, {c, d}, {a, b, c, d}, X} Then $(1, 2)^*$-πgη-O(X) = {ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {b, c, d}, {c, d, e}, {a, b, c, d}, {a, b, d, e}, {a, c, d, e}, {b, c, d, e}}. The set {a, d} is $(1, 2)^*$-πgη-nbd of the point a, there exists an $(1, 2)^*$-πgη-open set {a} is such that $a \in \{a\} \subset \{a, d\}$. However, the set {a, d} is not a nbd of the point a, since no open set G exists such that $a \in G \subset \{a, d\}$.

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Theorem 4.21. If a subset N of a space $(X, \Im_1, \Im_2)$ is $(1, 2)^*$-πgη-open, then N is a $(1, 2)^*$-πgη-nbd of each of its points.
Proof. Suppose N is $(1, 2)^*$-πgη-open. Let $x \in N$. We claim that N is $(1, 2)^*$-πgη-nbd of x. For N is a $(1, 2)^*$-πgη-open set such that $x \in N \subset N$. Since x is an arbitrary point of N, it follows that N is a $(1, 2)^*$-πgη-nbd of each of its points.

Definition 4.22. Let x be a point in a space $(X, \Im_1, \Im_2)$. The set of all $(1, 2)^*$-πgη-nbhd of x is called the **$(1, 2)^*$-πgη-nbd system** at x, and is denoted by $(1, 2)^*$-πgη-N(x).

Theorem 4.23. Let $(X, \Im_1, \Im_2)$ be a bitopological space and for each $x \in (X, \Im_1, \Im_2)$. Let $(1, 2)^*$- πgη-N(x) be the collection of all $(1, 2)^*$-πgη-nbds of x. Then we have the following results.
(i) $\vee\, x \in (X, \Im_1, \Im_2)$, $(1, 2)^*$-πgη-N(x) $\neq \phi$.
(ii) $N \in (1, 2)^*$-πgη-N(x) $\Rightarrow x \in N$.
(iii) $N \in (1, 2)^*$-πgη-N(x), $M \supset N \Rightarrow M \in (1, 2)^*$-πgη-N(x).
(iv) $N \in (1, 2)^*$-πgη-N(x), $M \in (1, 2)^*$-πgη-N(x) $\Rightarrow N \cap M \in (1, 2)^*$-πgη-N(x).
(v) $N \in (1, 2)^*$-πgη-N(x) $\Rightarrow$ there exists $M \in (1, 2)^*$-πgη-N(x) such that $M \subset N$ and $M \in (1, 2)^*$- πgη-N(y) for every $y \in M$.
Proof. (i) Since $(X, \Im_1, \Im_2)$ is a $(1, 2)^*$-πgη-open set, it is a $(1, 2)^*$-πgη-nbd of every $x \in (X, \Im_1, \Im_2)$. Hence there exists at least one $(1, 2)^*$-πgη-nbd (namely - $(X, \Im_1, \Im_2)$) for each $x \in (X, \Im_1, \Im_2)$. Hence $(1, 2)^*$-πgη-N(x) $\neq \phi$ for every $x \in (X, \Im_1, \Im_2)$.

(ii) If $N \in (1, 2)^*$-πgη-N(x), then N is a $(1, 2)^*$-πgη-nbd of x. So by definition of $(1, 2)^*$-πgη-nbd, $x \in N$.

(iii) Let $N \in (1, 2)^*$-πgη-N(x) and $M \supset N$. Then there is a $(1, 2)^*$-πgη-open set G such that $x \in G \subset N$. Since $N \subset M$, $x \in G \subset M$ and so M is $(1, 2)^*$-πgη-nbd of x. Hence $M \in (1, 2)^*$-πgη-N (x).

(iv) Let $N \in (1, 2)^*$-πgη-N(x) and $M \in (1, 2)^*$-πgη-N(x). Then by definition of $(1, 2)^*$-rgη-nbd $x \in G_1 \cap G_2 \subset N \cap M \Rightarrow$ (i). Since $G_1 \cap G_2$ is a $(1, 2)^*$-πgη-open set, (being the intersection of two $(1, 2)^*$-πgη-open sets), it follows from (i) that $N \cap M$ is a $(1, 2)^*$-πgη-nbd of x. Hence $N \cap M \in (1, 2)^*$-πgη-N(x).

(v) If $N \in (1, 2)^*$-πgη-N(x), then there exists a $(1, 2)^*$-πgη-open set M such that $x \in M \subset N$. Since M is a $(1, 2)^*$-πgη-open set, it is $(1, 2)^*$-πgη-nbd of each of its points. Therefore $M \in (1, 2)^*$-πgη-N(y) for every $y \in M$.

5. SOME $(1, 2)^*$-πGη-BITOPOLOGICAL FUNCTIONS

In this section, we shall recall the definitions of some functions used in the sequel. Further we introduce some $(1, 2)^*$-πgη-continuous functions in bitopological spaces.

Definition 5.1. A function $f : X \to Y$ is said to be
(i) **$(1, 2)^*$-η-continuous** [**9**] if $f^{-1}(F)$ is $(1, 2)^*$-η-closed in X for every $\Im_{1,2}$-closed set F of Y;
(ii) **$(1, 2)^*$-gη-continuous** [**9**] if $f^{-1}(F)$ is $(1, 2)^*$-gη-closed in X for every $\Im_{1,2}$-closed set F of Y;
(iii) **$(1, 2)^*$-πgη-continuous** if $f^{-1}(F)$ is $(1, 2)^*$-πgη-closed in X for every $\Im_{1,2}$-closed set F of Y;
(iv) **$(1, 2)^*$-rgη-continuous** [**9**] if $f^{-1}(F)$ is $(1, 2)^*$-rgη-closed in X for every $\Im_{1,2}$-closed set F of Y;
(v) **$(1, 2)^*$-R-map** [**4**] if $f^{-1}(F) \in (1, 2)^*$-RO(X) for every $F \in (1, 2)^*$-RO(Y);
(vi) **completely $(1, 2)^*$-continuous** [**4**] if $f^{-1}(F) \in (1, 2)^*$-RO(X) for every $\Im_{1,2}$-open set F of Y.
(vii) **$(1, 2)^*$-π-continuous** [**2**] if $f^{-1}(F)$ is $(1, 2)^*$-π-closed in X for every $\Im_{1,2}$-closed set F of Y;

Definition 5.2. A function $f : X \to Y$ is said to be
(i) **almost $(1, 2)^*$-continuous** [**4**] if $f^{-1}(F)$ is $\Im_{1,2}$-open in X for every $F \in (1, 2)^*$-RO(Y);
(ii) **almost $(1, 2)^*$-π-continuous** [**2**] if $f^{-1}(F)$ is $(1, 2)^*$-π-closed in X for every $F \in (1, 2)^*$-RC(Y);
(iii) **almost $(1, 2)^*$-η-continuous** [**9**] if $f^{-1}(F)$ is $(1, 2)^*$-η-closed in X for every $F \in (1, 2)^*$-RC(Y);
(iv) **almost $(1, 2)^*$-gη-continuous** [**9**] if $f^{-1}(F)$ is $(1, 2)^*$-gη-closed in X for every $F \in (1, 2)^*$-RC(Y);
(v) **almost $(1, 2)^*$-πgη-continuous** if $f^{-1}(F)$ is $(1, 2)^*$-πgη-closed in X for every $F \in (1, 2)^*$-RC(Y);
(vi) **almost $(1, 2)^*$-rgη-continuous** [**9**] if $f^{-1}(F)$ is $(1, 2)^*$-rgη-closed in X for every $F \in (1, 2)^*$-RC(Y);

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Remark 5.3. From the definitions stated above, we obtain the following diagram:

$$\begin{array}{ccc}
\text{complete } (1,2)^{*}\text{-continuity} & \Rightarrow & (1,2)^{*}\text{-R-map} \\
\Downarrow & & \Downarrow \\
(1,2)^{*}\text{-}\pi\text{-continuity} & \Rightarrow & \text{almost } (1,2)^{*}\text{-}\pi\text{-continuity} \\
\Downarrow & & \Downarrow \\
(1,2)^{*}\text{-continuity} & \Rightarrow & \text{almost } (1,2)^{*}\text{-continuity} \\
\Downarrow & & \Downarrow \\
(1,2)^{*}\text{-}\eta\text{-continuity} & \Rightarrow & \text{almost } (1,2)^{*}\text{-}\eta\text{-continuity} \\
\Downarrow & & \Downarrow \\
(1,2)^{*}\text{-g}\eta\text{-continuity} & \Rightarrow & \text{almost } (1,2)^{*}\text{-g}\eta\text{-continuity} \\
\Downarrow & & \Downarrow \\
(1,2)^{*}\text{-}\pi\text{g}\eta\text{-continuity} & \Rightarrow & \text{almost } (1,2)^{*}\text{-}\pi\text{g}\eta\text{-continuity} \\
\Downarrow & & \Downarrow \\
(1,2)^{*}\text{-rg}\eta\text{ -continuity} & \Rightarrow & \text{almost } (1,2)^{*}\text{-rg}\eta\text{-continuity}
\end{array}$$

The following examples enable us to realize that none of the implications in the above diagram is reversible.

Example 5.4. Let X = {a, b, c}, $\Im_1$ = {ϕ, X, {a}, {a, c}} and $\Im_2$ = {ϕ, X, {c}}. Let Y = {a, b, c}, ψ_1 = {ϕ, Y, {a}} and ψ_2 = {ϕ, Y, {b}}. Define f : X → Y as f(a) = c; f(b) = b; f(c) = a. Clearly f is almost $(1, 2)^*$-rgη-continuous but not almost $(1, 2)^*$-πgη-continuous.

Example 5.5. Let X = {a, b, c}, $\Im_1$ = {ϕ, X, {a}, {a, c}} and $\Im_2$ = {ϕ, X, {c}}. Let Y = {a, b, c}, ψ_1 = {ϕ, Y, {a}} and ψ_2 = {ϕ, Y, {b}}. Define f : X → Y as f(a) = a, f(b) = c, f(c) = b. Clearly f is $(1, 2)^*$-continuous as well as $(1, 2)^*$-rgη-continuous but it is not completely $(1, 2)^*$-continuous.

Example 5.6. Let X = Y = {a, b, c}, $\Im_1$ = {ϕ, X, {a}} and $\Im_2$ = {ϕ, X, {a, b}}. Let ψ_1 = {ϕ, Y, {a}} and ψ_1 = {ϕ, Y, {a, c}}. Define f : X → Y as f(a) = b; f(b) = c; f(c) = a. Clearly f is both $(1, 2)^*$-πgη-continuous and almost $(1, 2)^*$-πgη-continuous but it is neither $(1, 2)^*$-η-continuous nor $(1, 2)^*$-gη-continuous. It is not $(1, 2)^*$-continuous

Example 5.7. Let X = Y = {a, b, c}, $\Im_1$ = {ϕ, X, {a}} and $\Im_2$ = {ϕ, X, {a, b}}. Let ψ_1 = {ϕ, Y, {a}} and ψ_1 = {ϕ, Y, {a, c}}. Define f : X → Y as f(a) = c, f(b) = a, f(c) = b. Clearly f is both $(1, 2)^*$-πgη-continuous and almost $(1, 2)^*$-πgη-continuous. But it is neither $(1, 2)^*$-continuous nor $(1, 2)^*$-η-continuous.

Example 5.8. Let X = Y = {a, b, c}, $\Im_1$ = {ϕ, X, {a}} and $\Im_2$ = {ϕ, X, {b}, {a, b}}. Let ψ_1 = {ϕ, Y, {a}} and ψ_2 = {ϕ, Y, {b}}. Define f : X → Y as f(a) = b; f(b) = a; f(c) = c. Clearly f is almost $(1, 2)^*$-continuous as well as almost $(1, 2)^*$-πgη-continuous.

Example 5.9. Let X = Y = {a, b, c}, $\Im_1$ = {ϕ, X, {a}} and $\Im_2$ = {ϕ, X, {b, c}}. Let ψ_1 = {ϕ, Y, {b}, {c}, {b, c}} and ψ_2 = {ϕ, Y, {a, b}}. Define f : X → Y as f(a) = a; f(b) = c; f(c) = b. Clearly f is almost $(1, 2)^*$-gη-continuous as well as almost $(1, 2)^*$-πgη-continuous but it is neither almost $(1, 2)^*$-continuous nor almost $(1, 2)^*$-η-continuous. It is neither $(1, 2)^*$-continuous nor $(1, 2)^*$-η-continuous.

Definition 5.10. A space X is said to be **$(1, 2)^*$-πgη-$T_{1/2}$** if every $(1, 2)^*$-πgη-closed set of X is $(1, 2)^*$-π-closed in X.

Definition 5.11. A function f : X → Y is said to be **$(1, 2)^*$-πgη-irresolute** if $f^{-1}(F)$ is $(1, 2)^*$- πgη-closed in X for every $(1, 2)^*$-πgη-closed set F of Y .

6. CONCLUSION

In this paper, we introduce and investigate a new class of sets called $(1, 2)^*$-πgη-closed and some new functions called $(1, 2)^*$-πgη-continuous, almost $(1, 2)^*$-πgη-continuous functions in bitopological spaces. Moreover we obtain the relationships among some existing closed sets like $(1, 2)^*$-πgη-closed, $(1, 2)^*$-semi-closed, $(1, 2)^*$-α-closed, $(1, 2)^*$-η-closed sets and their generalizations. Also we study some basic properties of $(1, 2)^*$-πgη-closed sets. Further, we introduce $(1, 2)^*$-πgη-neighbourhood and discuss some basic properties of $(1, 2)^*$-πgη-neighbourhood. The $(1, 2)^*$-πgη-closed sets can be used to derive a new decomposition of unity, closed map and open map, homeomorphism, closure and interior and new separation axioms. This idea can be extended to ordered topological, ordered bitopological and fuzzy topological spaces etc.

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